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A note on the New Optimization Model for Traffic Problem

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1 Introduction

The purpose of this note is twofold: (1) to give the background in the paper by Gu et al. (2016) on the tri-level optimization model for private road competition problem with traffic equilibrium constraints, and (2) to summarize the main idea and model in that paper.

In recent years, due to the funds limitation, there has been a growing trend for governments to allow private participation in some major public investments, especially for infrastructure projects, such as expressways and highway, under a procurement system called build-operate-transfer (BOT) (Roth, 1996). Under this scheme, the project sponsor is responsible for financing, construction and operating roads, and in return, should receive the revenue from road toll charge for some years. After that, the roads will be transferred to the government. The major motivation for the BOT scheme is that the government does not need to spend any public funding while the public infrastructure can still be constructed.

Meanwhile, the private firms can enjoy a high potential profit from a successful BOT project. There are many research works in the literature discussing the problem of public-private-partnership in transportation infrastructure construction. Given an existing transport network system, to alleviate the traffic congestion, the government decides to develop certain number of new roads. However, the government has limited fund and can not afford to invest in building up all of these roads, so the private firms are encouraged to participate the development of these new roads. Yang et al. (2009) considered a toll road competition under a traffic network of BOT. There are two or more different participating private firms and the firms can make their own optimal decisions on both the investment (capacity) and toll charges. The equilibrium problem is solved based on the assumption that the private firms determine their operation strategies on multiple toll roads in a road network to maximize their profit. However, if all the new roads are built and operated by private firms under the BOT scheme, as was assumed in Yang et al. (2009), the primary objectives of all the private firms are to maximize their profits, which is deviated from the original target of traffic congestion mitigation from government's perspective.

Noting that only private participation in the road competition problem may lead to a less efficient system and result in a traffic network performance that is deviated from the desired social welfare maximization case, it is imperative for the government to understand what role the government should play in the situation of commercialization and private supply of road and how the government should determine the optimal participation strategy ensuring the proper usage of private road. In Yang et al. (2009), the government played a less important role in the system: not being able to participate the new road construction and operation, the government can only use regulations to manage the system, which would be much less efficient in achieving the goal of maximizing social welfare of the system.

If government controls some tolled roads, it is possible to influence the operation strategies of private roads to avoid the reduced social welfare, and at the same time guarantee the private enterprises can gain the profit. Obviously, the answer to these questions and issues remain unclear and will be addressed in this study.

In the paper (Gu and Cai, 2016), we study the problem that how the government should take part in the road competition while taking into consideration of the private firms' responsive strategies and the travelers' equilibrium travel pattern. It is assumed that the government plans to construct certain number of new roads to improve the transport network performance. While most of the new roads would be built up by the private firms under BOT scheme, the government itself would also participate in the road construction and operation on certain new roads. By doing so, it will be more efficient for the government to achieve the goal of best managing the network traffic and maximizing the social welfare, as compared to the case of letting all the new roads constructed and operated by only profit-maximizing private firms under BOT scheme. Because of the target is that basing on getting the social welfare to make the firms compete to each other by themselves. The government doesn't compete with the private firms. It plays as the guide and wants to lead to more invest from the firms for the roads while doesn't let them to bid up price. Therefor, the one-shot game and Stackelberg game is not exact suitable for our model.

We proposed a tri-level mathematical programming model formulate this problem and propose a heuristic algorithm to solve this model. The upper level program determines the optimal toll to maximize the social welfare.

The middle level describes the private firms' optimal strategies on their investment (road capacities) and toll charge that maximizes their profits in response to the government's road toll decision.

In the lower level, the user equilibrium traffic assignment is conducted, assuming all the road users make route choices following the deterministic user equilibrium principle (Wardrop, 1952). And we formulate it as a finite-dimensional variational inequality and solve it by projection-type method with BB step size (He et al., 2012).

2 Traffic Network Description

We consider a directed transportation network $G = (N, A)$, consisting of a set N of nodes and a set A of links whose elements are ordered pairs of distinct nodes.

The following are the notations used in this paper:

v_a : the flow on link $a \in A$, and $\boldsymbol{v} = (v_a : a \in A)^T$ be the vectors of link flows,

t_a: an associated flow-dependent cost to a, is assumed to be differentiable and monotonically increasing with the amount of flow v_a, and $\boldsymbol{t}(\boldsymbol{v}) = (t_a(v_a) : a \in A)^T$ be the vector of link travel cost,

W : the set of Origin-Destination (OD) pairs,

R_w : the set of all paths connecting OD pair $w \in W$, and $R = \bigcup_{w \in W} R_w$,

d_w : the traffic demand traveling between OD pair $w \in W$, and $\boldsymbol{d} = (d_w : w \in W)^T$ be the vectors of OD demands,

f_{rw} : the flow on path $r \in R_w$ of OD pair $w \in W$, and $\boldsymbol{f} = (f_{rw} : r \in R_w, w \in W)^T$ be the vectors of all path flows,

c_{rw} : the travel cost along a path $r \in R_w$ of OD pair $w \in W$, is the sum of travel costs on all links that comprise the path, i.e, $c_{rw} = \sum_{a \in r} t_a(v_a)$, and $\boldsymbol{c} = (c_{rw} : r \in R_w, w \in W)^T$ be the column vectors of path travel cost,

μ_w : the minimum path cost of OD pair $w \in W$, defined as $\mu_w = \min\{c_{rw} : \ r \in R_w\}$, and $\boldsymbol{\mu} = (\mu_w : w \in W)^T$,

Δ : $\Delta = [\Delta_{ar}]$ be the link-path incidence matrix, where Δ_{ar} equals 1 if path r includes link a and 0 otherwise,

Λ : $\Lambda = [\Lambda_{wr}]$ be the OD-path incidence matrix, where Λ_{wr} equals 1 if path r connects OD pair w and 0 otherwise.

3 Model Formulation

The Whole Model: The problem under consideration can be formulated as:

$$\text{(Upper level)} \quad \max \quad W(\tau) = \left(\sum_{w \in W} \int_0^{d_w} B_w(\omega)\mathrm{d}\omega - \sum_{a \in A} v_a t(v_a) - \sum_{a \in J, a \in K} \frac{1}{\beta}\eta I_a \right) - \rho \sum_{i \in K} \frac{1}{\beta}\tau_i v_i \quad (1)$$

$$\text{s.t.} \quad \tau_i v_i \geq \eta I_i, \; i \in K, \quad (2)$$

$$\text{(Middle level)} \quad (u_j, y_j) \in S^j_{l_2}, \; j \in J, \quad (3)$$

$$\text{(Lower level)} \quad (\boldsymbol{v}, \boldsymbol{d}) \in S_{l_1}. \quad (4)$$

Note that A is the set of all links in the network and $K \subseteq A, J \subseteq A$.

4 Heuristic Algorithm

Similar to the method in solving the EPEC problem, here we propose a heuristic method for the computation of the tri-level optimization problem, a synchronous iterative method.

The user equilibrium problem can be formulated as a finite-dimensional Variational Inequality (VI) (Facchinei, 2003). We describe the reformulated VI formally and then adopt the recently developed projection-type method with BB step size to solve it (He et al. 2012).

From practical point of view, the government should adjust the level of its tolls as less as possible. Therefore we can control the upper iteration number for some suitable figure. While on the other hand, private companies can make appropriate adjustments according to the information of the government's toll level and the users' choice, and choose their own optimal strategies.

We have to point out that the study of this tri-level optimization is still in its infancy, and the computation of a global solution is difficult, if not impossible; this can be observed from the special case of solving Mathematical Programs with Equilibrium Constraints (MPEC), i.e., the government does not involve in the system. The issue of convergence of the heuristic methods will be addressed in the future study.

5 Conclusions

In this paper, we studied the situation in which the government, devoting to maximize the social welfare, and the private company, urging for maximum profit, exist in the same road network and operate different tolled roads simultaneously. Envisaging that the participation of government in new road construction alongside with private firms' BOT scheme would better manage the network traffic and achieve the government's goal of maximizing social welfare, we analyzed the interaction relationship of pricing, road capacity and competition and how the road capacity and price are settled by using game theoretical model.

We develop a tri-level model to describe the problem. A heuristic solution algorithm is proposed to solve the model. The mode results indicate that the model is meaningful as the fundamental role for the government is to make increase in social welfare.

References

[1] Facchinei, F. and Pang, J.S., 2003. Finite-Dimensional Variational Inequalities and Complementarity Problems. Springer-Verlag.

[2] Guo, X. L. and Yang, H., 2009. Analysis of a build operate transfer scheme for road franchising. International Journal of Sustainable Transportation 3, 312-338.

[3] He, H., Han, D. and Li, Z., 2012. Some projection methods with the BB step sizes for variational inequalities. Journal of Computational and Applied Mathematics 236, 2590-2604.

[4] Roth, G., 1996. Roads in a market economy. Avebury Technical, Ashgate Publishing, UK, England.

[5] Wardrop, J.G., 1952. Some theoretical aspects of road traffic research. Proceedings of the Institute of Civil Engineers, Pt. II 1, 325-378.

[6] Yang, H., Xiao, F. and Huang, H., 2009. Private road competition and equilibrium with traffic equilibrium constraints. Journal of Advanced Transportation 36, 169-185.

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